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Global stability and Sensitivity Analysis of Malaria, Dengue and Typhoid Triple Infection

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ABSTRACT: This study investigates the global stability of the endemic equilibrium point of triple co-infection of malaria, dengue, and typhoid. By using an appropriate Lyapunov function, the results show that the model is globally asymptotically stable. This implies that the diseases can be eradicated or kept at low levels, regardless of the population. Sensitivity analysis was also conducted to identify the most sensitive parameter. The results indicate that strategies to reduce malaria and dengue fever vectors should be prioritized to curb the spread of the diseases. Additionally, minimizing exposure to contaminated water and food, as well as reducing the discharge of typhoid bacteria into the environment, can help to reduce or curb the spread of typhoid in the environment.

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Malaria is an infectious disease caused by the Plasmodium parasite and is transmitted through the bites of infected female Anopheles mosquitoes (Azuaba *et al.*, 2020; Ogunmiloro, 2019). According to the World Health Organization (WHO), malaria is responsible for an estimated 435,000 deaths annually, with 93% of these cases occurring in Africa. In contrast, dengue (DENV) is a viral disease that is transmitted to humans by infected female Aedes aegypti mosquitoes. The infection can range from mild illness to more severe forms such as dengue hemorrhagic fever (DHF) and dengue shock syndrome (DSS) (Otu *et al.*, 2019). Typhoid, on the other hand, is an infectious disease caused by the bacteria Salmonella Typhi and is spread through contaminated food and water (Atokolo and Omale, 2018). The signs and symptoms of typhoid fever include sustained fever, poor appetite, severe headache, fatigue, and vomiting. The incubation period for typhoid fever is typically between 7 and 14 days (Nthiiri, 2017). The triple infection of malaria, dengue, and typhoid has been reported in several cases and may lead to more severe symptoms and complications. This highlights the need for further research and effective prevention and treatment strategies for this potentially serious health concern. The concept of global stability is concerned with the global properties of a model, which can be investigated using Lyapunov function theory. The Lyapunov method has been effectively employed to demonstrate the global stability of the endemic equilibrium. This approach involves identifying a function, referred to as the Lyapunov function, that is

positive definite, with its derivative along the trajectories being negative (Atokolo *et al*., 2021).

Few works have been done to analyze the global stability of disease models (Nthiiri, 2017; Atokolo *et al*., 2021; Ogunmiloro, 2019; Kazeem *et al*., 2016; Bessey *et al.*, 2019; Bello *et al*., 2019; Peter *et al*., 2020). Oluwafemi *et al.* (2020) worked on the stability analysis of the disease free equilibrium of malaria, dengue and typhoid triple infection model to construct and analyze the global stability of the triple infection. This present work is therefore an extension of the Oluwafemi and co-workers research with an improvement to global stability and sensitivity analysis of Malaria, Dengue and Typhoid Triple Infection.

MATERIALS AND METHODS

Model Formulation: The model as proposed in (Oluwafemi *et al.*, 2020), divides the human population into Susceptible human S_h; Malaria infected human I_{hm}; Dengue Infected human I_{hd}; Typhoid infected human I_{ht}; co-infection of Malaria and Dengue I_{md}; Malaria and Typhoid I_{mt}; Dengue and Typhoid I_{dt}; Malaria, Dengue and Typhoid I_{mdt}; Recovered class R; the vector population is subdivided into; Non-disease carrier vector S_v; Malaria parasite vector carrier I_{vm}; Dengue virus vector carrier I_{vd} and the Typhoid carrier Bacteria W.

The susceptible human are recruited by a constant rate defined as Λ, they are infected by malaria, dengue and typhoid respectively at the rates $\alpha_{hm}, \alpha_{hd}, \alpha_{ht}$. The susceptible compartment is further reduced by the natural death rate μ_h and increased by the rate at which infected humans recovers δ. The malaria infected human compartment is increased by the rate at which individuals acquires malaria α_{hm}, rate at which individuals recovers from dengue fever when co-infected with malaria and dengue ρ_{hd} and recovery rate from typhoid when co-infected with typhoid and malaria ρ_{ht}. The compartment is reduced by natural death rate μ_h, malaria induced death rate η_{hm}, malaria only recovery rate ρ_{hm} and rate of acquiring dengue fever α_{hd} and typhoid α_{ht}.

The system of equations representing the transmission dynamics of the triple infection is presented as follows:

$$\frac{dS_h}{dt} = \Lambda + \delta R - (\alpha_{hm} + \alpha_{hd} + \alpha_{ht} + \mu_h)S_h, \quad (1)$$

$$\frac{dI_{hm}}{dt} = \alpha_{hm}S_h + \rho_{hd}I_{md} + \rho_{ht}I_{mt} - (\alpha_{hd} + \alpha_{ht} + \rho_{hm} + \eta_{hm} + \mu_h)I_{hm}, \quad (2)$$

$$\frac{dI_{hd}}{dt} = \alpha_{hd}S_h + \rho_{hm}I_{md} + \rho_{ht}I_{dt} - (\alpha_{hm} + \alpha_{ht} + \rho_{hd} + \eta_{hd} + \mu_h)I_{hd}, \quad (3)$$

$$\frac{dI_{ht}}{dt} = \alpha_{ht}S_h + \rho_{hm}I_{mt} + \rho_{hd}I_{dt} - (\alpha_{hm} + \alpha_{hd} + \rho_{ht} + \eta_{ht} + \sigma_1 + \mu_h)I_{ht}, \quad (4)$$

$$\frac{dI_{md}}{dt} = \alpha_{hm}I_{hd} + \alpha_{hd}I_{hm} + \rho_{ht}I_{mdt} - (\alpha_{ht} + \rho_{hd} + \rho_{hm} + \eta_{hm} + \eta_{hd} + \mu_h)I_{md}, \quad (5)$$

$$\frac{dI_{mt}}{dt} = \alpha_{hm}I_{ht} + \alpha_{ht}I_{hm} + \rho_{hd}I_{mdt} - (\alpha_{hd} + \rho_{ht} + \rho_{hm} + \eta_{hm} + \eta_{ht} + \sigma_2 + \mu_h)I_{mt}, \quad (6)$$

$$\frac{dI_{dt}}{dt} = \alpha_{ht}I_{hd} + \alpha_{hd}I_{ht} + \rho_{hm}I_{mdt} - (\alpha_{hm} + \rho_{hd} + \rho_{ht} + \eta_{ht} + \eta_{hd} + \sigma_3 + \mu_h)I_{dt}, \quad (7)$$

$$\frac{dI_{mdt}}{dt} = \alpha_{ht}I_{md} + \alpha_{hd}I_{mt} + \alpha_{hm}I_{dt} - \begin{pmatrix}\rho_{hm} + \rho_{hd} + \rho_{ht} + \eta_{hm} \\ +\eta_{ht} + \eta_{hd} + \sigma_4 + \mu_h\end{pmatrix} I_{mdt}, \quad (8)$$

$$\frac{dR}{dt} = \rho_{hm}I_{hm} + \rho_{hd}I_{hd} + \rho_{ht}I_{ht} - \delta R, \quad (9)$$

$$\frac{dS_v}{dt} = \Lambda_v - (\beta_1 + \beta_2 + \mu_v)S_v, \quad (10)$$

$$\frac{dI_{vm}}{dt} = \beta_1 S_v - \mu_v I_{vm}, \quad (11)$$

$$\frac{dI_{vd}}{dt} = \beta_2 S_v - \mu_v I_{vd}, \quad (12)$$

$$\frac{dW}{dt} = \sigma_1 I_{ht} + \sigma_2 I_{mt} + \sigma_3 I_{dt} + \sigma_4 I_{mdt} - \mu_b W. \quad (13)$$

where

$$\left.\begin{aligned} \alpha_{hm} = \frac{b_m \vartheta_m I_{vm}}{N_h};\ \alpha_{hd} = \frac{b_d \vartheta_d I_{vd}}{N_h};\ \alpha_{ht} = vW; \\ \beta_1 = \frac{b_m \vartheta_m (I_{hm}+I_{md}+I_{mt}+I_{mdt})}{N_h};\ \beta_2 = \frac{b_d \vartheta_d (I_{hd}+I_{md}+I_{dt}+I_{mdt})}{N_h}. \end{aligned}\right\} \quad (14)$$

The description of the variables and parameters of the model can be found in Tables 1 and 2, respectively.

Table 1. Model Variables

Variables	Description
S_h	Susceptible human
I_{hm}	Malaria infected human
I_{hd}	Dengue Infected human
I_{ht}	Typhoid infected human
I_{md}	Malaria and Dengue Co-infection
I_{mt}	Malaria and Typhoid Co-infection
I_{dt}	Dengue and Typhoid Co-infection
I_{mdt}	Malaria, Dengue, and Typhoid co-infection
R	Recovered human
S_v	Non-disease carrier vector
I_{vm}	Malaria parasite vector carrier
I_{vd}	Dengue virus vector carrier
W	Typhoid carrier Bacteria

Table 2. Model Parameters

Parameter	Description
Λ	Human Recruitment rate
δ	Rate at which recovered become susceptible
ρ_{hm}	Recovery rate for malaria only
ρ_{hd}	Recovery rate for dengue fever only
ρ_{ht}	Recovery rate for typhoid only
α_{hm}	Rate at which one acquires malaria
α_{hd}	Rate at which one acquires dengue
α_{ht}	Rate at which one acquires typhoid
η_{hm}	Malaria induced death
η_{hd}	Dengue induced death
η_{ht}	Typhoid induced death
μ_h	Human Natural death rate
Λ_v	Vector recruitment rate
μ_v	Vector natural death
σ	Typhoid Bacteria discharge rate
v	Rate of exposure to contaminated food or water
μ_b	Bacteria death rate
b_m	Probability of transmission of malaria
b_d	Probability of transmission of dengue
ϑ_m	Number of bites of malaria carrier vector per time
ϑ_d	Number of bites of dengue carrier vector per time

Equilibria Points: This model has the Disease Free Equilibrium (DFE) and Endemic Equilibrium (EE). At DFE, there are no infections, hence

$$I_{hm} = I_{hd} = I_{ht} = I_{md} = I_{mt} = I_{dt} = I_{mdt} = I_{vm} = I_{vd} = 0. \quad (15)$$

Hence DFE is given as

$$\begin{aligned} &\varepsilon^0 \\ &= (S_h, I_{hm}, I_{hd}, I_{ht}, I_{md}, I_{mt}, I_{dt}, I_{mdt}, R, S_v, I_{vm}, I_{vd}, W) \\ &= \left(\frac{\Lambda}{\mu_h}, 0,0,0,0,0,0,0,0, \frac{\Lambda_v}{\mu_v}, 0,0,0\right). \end{aligned} \quad (16)$$

The EE is the point where there are infections. Computing the EE point is complex.

Basic Reproduction Number: The basic reproduction number is the average number of secondary infections produced by a single infected individual in a susceptible population. The computation of the Basic Reproduction Number involves applying the next-generation method. This method defines the basic reproduction number as the maximum value of the spectral radius of the matrix FV^{-1}, where F and V represent the rates of appearance of new infections and the rate of movement in or out of a compartment, respectively. The matrices at DFE are given by

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_4 & 0 & 0 & a_4 & a_4 & 0 & a_4 & 0 & 0 & 0 \\ 0 & a_5 & 0 & a_5 & 0 & a_5 & a_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (17)$$

$$V = \begin{pmatrix} u_1 & 0 & 0 & -\rho_{hd} & -\rho_{ht} & 0 & 0 & 0 & 0 & 0 \\ 0 & u_2 & 0 & -\rho_{hm} & 0 & -\rho_{ht} & 0 & 0 & 0 & 0 \\ 0 & 0 & u_3 & 0 & -\rho_{hm} & -\rho_{hd} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & u_4 & 0 & 0 & -\rho_{ht} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & u_5 & 0 & -\rho_{hd} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & u_6 & -\rho_{hm} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & u_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_v & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_v & 0 \\ 0 & 0 & -\sigma_1 & 0 & -\sigma_2 & -\sigma_3 & -\sigma_4 & 0 & 0 & \mu_b \end{pmatrix}. \quad (18)$$

Where

$$a_1 = b_m \vartheta_m; a_2 = b_d \vartheta_d;\ a_3 = vS_h;\ a_4 = \frac{b_m \vartheta_m S_v}{N_h};\ a_5 = \frac{b_d \vartheta_d S_v}{N_h}, \quad (19)$$

$$u_1 = (\rho_{hm} + \eta_{hm} + \mu_h);\ u_2 = (\rho_{hd} + \eta_{hd} + \mu_h); u_3 = (\rho_{ht} + \eta_{ht} + \sigma_1 + \mu_h), \quad (20)$$

$$u_4 = (\rho_{hd} + \rho_{hm} + \eta_{hm} + \eta_{hd} + \mu_h); u_5 = (\rho_{ht} + \rho_{hm} + \eta_{hm} + \eta_{ht} + \sigma_2 + \mu_h), \quad (21)$$

$$\left.\begin{aligned} u_6 &= (\rho_{hd} + \rho_{ht} + \eta_{ht} + \eta_{hd} + \sigma_3 + \mu_h), \\ u_7 &= (\rho_{hm} + \rho_{hd} + \rho_{ht} + \eta_{hm} + \eta_{ht} + \eta_{hd} + \sigma_4 + \mu_h). \end{aligned}\right\} \quad (22)$$

The basic reproduction number is given as

$$R_0 = \max\left\{\sqrt{\frac{\Lambda_v \mu_h b_m^2 \vartheta_m^2}{(\rho_{hm}+\eta_{hm}+\mu_h)\Lambda\mu_v^2}}, \sqrt{\frac{\Lambda_v \mu_h b_d^2 \vartheta_d^2}{(\rho_{hd}+\eta_{hd}+\mu_h)\Lambda\mu_v^2}}, \frac{v\Lambda\sigma_1}{(\rho_{ht}+\eta_{ht}+\sigma_1+\mu_h)\mu_b\mu_h}\right\}. \quad (23)$$

Global Stability Analysis

Theorem 1: The endemic equilibrium $\mathcal{E}_E$ of the system is globally asymptotically stable wherever $R_0 > 1$.

Proof: We construct a Lyapunov function

$$V(S_h^*, I_{hm}^*, I_{hd}^*, I_{ht}^*, I_{md}^*, I_{mt}^*, I_{dt}^*, I_{mdt}^*, R^*, S_v^*, I_{vm}^*, I_{vd}^*, W^*) =$$
$$\begin{bmatrix} \left(S_h - S_h^* - S_h^* \log\frac{S_h^*}{S_h}\right) + \left(I_{hm} - I_{hm}^* - I_{hm}^* \log\frac{I_{hm}^*}{I_{hm}}\right) + \left(I_{hd} - I_{hd}^* - I_{hd}^* \log\frac{I_{hd}^*}{I_{hd}}\right) + \\ \left(I_{ht} - I_{ht}^* - I_{ht}^* \log\frac{I_{ht}^*}{I_{ht}}\right) + \left(I_{md} - I_{md}^* - I_{md}^* \log\frac{I_{md}^*}{I_{md}}\right) + \left(I_{mt} - I_{mt}^* - I_{mt}^* \log\frac{I_{mt}^*}{I_{mt}}\right) + \\ \left(I_{dt} - I_{dt}^* - I_{dt}^* \log\frac{I_{dt}^*}{I_{dt}}\right) + \left(I_{mdt} - I_{mdt}^* - I_{mdt}^* \log\frac{I_{mdt}^*}{I_{mdt}}\right) + \left(R - R^* - R^* \log\frac{R^*}{R}\right) + \\ \left(S_v - S_v^* - S_v^* \log\frac{S_v^*}{S_v}\right) + \left(I_{vm} - I_{vm}^* - I_{vm}^* \log\frac{I_{vm}^*}{I_{vm}}\right) + \left(I_{vd} - I_{vd}^* - I_{vd}^* \log\frac{I_{vd}^*}{I_{vd}}\right) + \\ \left(W - W^* - W^* \log\frac{W^*}{W}\right) \end{bmatrix}, \quad (24)$$

Differentiating we have

$$\frac{dV}{dt} = \left[\left(\frac{S_h - S_h^*}{S_h}\right)\frac{dS_h}{dt} + \left(\frac{I_{hm} - I_{hm}^*}{I_{hm}}\right)\frac{dI_{hm}}{dt} + \left(\frac{I_{hd} - I_{hd}^*}{I_{hd}}\right)\frac{dI_{hd}}{dt} + \left(\frac{I_{ht} - I_{ht}^*}{I_{ht}}\right)\frac{dI_{ht}}{dt} + \left(\frac{I_{md} - I_{md}^*}{I_{md}}\right)\frac{dI_{md}}{dt} + \left(\frac{I_{mt} - I_{mt}^*}{I_{mt}}\right)\frac{dI_{mt}}{dt} + \left(\frac{I_{dt} - I_{dt}^*}{I_{dt}}\right)\frac{dI_{dt}}{dt} + \left(\frac{I_{mdt} - I_{mdt}^*}{I_{mdt}}\right)\frac{dI_{mdt}}{dt} + \left(\frac{R - R^*}{R}\right)\frac{dR}{dt} + \left(\frac{S_v - S_v^*}{S_v}\right)\frac{dS_v}{dt} + \left(\frac{I_{vm} - I_{vm}^*}{I_{vm}}\right)\frac{dI_{vm}}{dt} + \left(\frac{I_{vd} - I_{vd}^*}{I_{vd}}\right)\frac{dI_{vd}}{dt} + \left(\frac{W - W^*}{W}\right)\frac{dW}{dt}\right]. \quad (25)$$

Substituting (1) we have

$$\frac{dV}{dt} = \left[\left(\frac{S_h - S_h^*}{S_h}\right)(\Lambda + \delta R - (\alpha_{hm} + \alpha_{hd} + \alpha_{ht} + \mu_h)S_h) + \left(\frac{I_{hm} - I_{hm}^*}{I_{hm}}\right)(\alpha_{hm}S_h + \rho_{hd}I_{md} + \rho_{ht}I_{mt} - (\alpha_{hd} + \alpha_{ht} + \rho_{hm} + \eta_{hm} + \mu_h)I_{hm}) + \left(\frac{I_{hd} - I_{hd}^*}{I_{hd}}\right)(\alpha_{hd}S_h + \rho_{hm}I_{md} + \rho_{ht}I_{dt} - (\alpha_{hm} + \alpha_{ht} + \rho_{hd} + \eta_{hd} + \mu_h)I_{hd}) + \left(\frac{I_{ht} - I_{ht}^*}{I_{ht}}\right)(\alpha_{ht}S_h + \rho_{hm}I_{mt} + \rho_{hd}I_{dt} - (\alpha_{hm} + \alpha_{hd} + \rho_{ht} + \eta_{ht} + \sigma_1 + \mu_h)I_{ht}) + \left(\frac{I_{md} - I_{md}^*}{I_{md}}\right)(\alpha_{hm}I_{hd} + \alpha_{hd}I_{hm} + \rho_{ht}I_{mdt} - (\alpha_{ht} + \rho_{hd} + \rho_{hm} + \eta_{hm} + \eta_{hd} + \mu_h)I_{md}) + \left(\frac{I_{mt} - I_{mt}^*}{I_{mt}}\right)(\alpha_{hm}I_{ht} + \alpha_{ht}I_{hm} + \rho_{hd}I_{mdt} - (\alpha_{hd} + \rho_{ht} + \rho_{hm} + \eta_{hm} + \eta_{ht} + \sigma_2 + \mu_h)I_{mt}) + \left(\frac{I_{dt} - I_{dt}^*}{I_{dt}}\right)(\alpha_{ht}I_{hd} + \alpha_{hd}I_{ht} + \rho_{hm}I_{mdt} - (\alpha_{hm} + \rho_{hd} + \rho_{ht} + \eta_{ht} + \eta_{hd} + \sigma_3 + \mu_h)I_{dt}) + \left(\frac{I_{mdt} - I_{mdt}^*}{I_{mdt}}\right)(\alpha_{ht}I_{md} + \alpha_{hd}I_{mt} + \alpha_{hm}I_{dt} - (\rho_{hm} + \rho_{hd} + \rho_{ht} + \eta_{hm} + \eta_{ht} + \eta_{hd} + \sigma_4 + \mu_h)I_{mdt}) + \left(\frac{R - R^*}{R}\right)(\rho_{hm}I_{hm} + \rho_{hd}I_{hd} + \rho_{ht}I_{ht} - \delta R) + \left(\frac{S_v - S_v^*}{S_v}\right)(\Lambda_v - (\beta_1 + \beta_2 + \mu_v)S_v) + \left(\frac{I_{vm} - I_{vm}^*}{I_{vm}}\right)(\beta_1 S_v - \mu_v I_{vm}) + \left(\frac{I_{vd} - I_{vd}^*}{I_{vd}}\right)(\beta_2 S_v - \mu_v I_{vd}) + \left(\frac{W - W^*}{W}\right)(\sigma_1 I_{ht} + \sigma_2 I_{mt} + \sigma_3 I_{dt} + \sigma_4 I_{mdt} - \mu_b W)\right]. \quad (26)$$

Collecting the positive and negative terms for the equation, we have

$$\frac{dV}{dt} = P_1 - P_2, \quad (27)$$

Where

$$P_1 = \left(\frac{S_h - S_h^*}{S_h}\right)(\Lambda + \delta R) + \left(\frac{I_{hm} - I_{hm}^*}{I_{hm}}\right)(\alpha_{hm} S_h + \rho_{hd} I_{md} + \rho_{ht} I_{mt}) + \left(\frac{I_{hd} - I_{hd}^*}{I_{hd}}\right)(\alpha_{hd} S_h + \rho_{hm} I_{md} + \rho_{ht} I_{dt}) + \left(\frac{I_{ht} - I_{ht}^*}{I_{ht}}\right)(\alpha_{ht} S_h + \rho_{hm} I_{mt} + \rho_{hd} I_{dt}) + \left(\frac{I_{md} - I_{md}^*}{I_{md}}\right)(\alpha_{hm} I_{hd} + \alpha_{hd} I_{hm} + \rho_{ht} I_{mdt}) + \left(\frac{I_{mt} - I_{mt}^*}{I_{mt}}\right)(\alpha_{hm} I_{ht} + \alpha_{ht} I_{hm} + \rho_{hd} I_{mdt}) + \left(\frac{I_{dt} - I_{dt}^*}{I_{dt}}\right)(\alpha_{ht} I_{hd} + \alpha_{hd} I_{ht} + \rho_{hm} I_{mdt}) + \left(\frac{I_{mdt} - I_{mdt}^*}{I_{mdt}}\right)(\alpha_{ht} I_{md} + \alpha_{hd} I_{mt} + \alpha_{hm} I_{dt}) + \left(\frac{R - R^*}{R}\right)(\rho_{hm} I_{hm} + \rho_{hd} I_{hd} + \rho_{ht} I_{ht}) + \left(\frac{S_v - S_v^*}{S_v}\right)(\Lambda_v) + \left(\frac{I_{vm} - I_{vm}^*}{I_{vm}}\right)(\beta_1 S_v) + \left(\frac{I_{vd} - I_{vd}^*}{I_{vd}}\right)(\beta_2 S_v) + \left(\frac{W - W^*}{W}\right)(\sigma_1 I_{ht} + \sigma_2 I_{mt} + \sigma_3 I_{dt} + \sigma_4 I_{mdt}), \quad (28)$$

and

$$P_2 = \left(\frac{S_h - S_h^*}{S_h}\right)\left((\alpha_{hm} + \alpha_{hd} + \alpha_{ht} + \mu_h) S_h\right) + \left(\frac{I_{hm} - I_{hm}^*}{I_{hm}}\right)\left((\alpha_{hd} + \alpha_{ht} + \rho_{hm} + \eta_{hm} + \mu_h) I_{hm}\right) + \left(\frac{I_{hd} - I_{hd}^*}{I_{hd}}\right)\left((\alpha_{hm} + \alpha_{ht} + \rho_{hd} + \eta_{hd} + \mu_h) I_{hd}\right) + \left(\frac{I_{ht} - I_{ht}^*}{I_{ht}}\right)\left((\alpha_{hm} + \alpha_{hd} + \rho_{ht} + \eta_{ht} + \sigma_1 + \mu_h) I_{ht}\right) + \left(\frac{I_{md} - I_{md}^*}{I_{md}}\right)\left((\alpha_{ht} + \rho_{hd} + \rho_{hm} + \eta_{hm} + \eta_{hd} + \mu_h) I_{md}\right) + \left(\frac{I_{mt} - I_{mt}^*}{I_{mt}}\right)\left((\alpha_{hd} + \rho_{ht} + \rho_{hm} + \eta_{hm} + \eta_{ht} + \sigma_2 + \mu_h) I_{mt}\right) + \left(\frac{I_{dt} - I_{dt}^*}{I_{dt}}\right)\left((\alpha_{hm} + \rho_{hd} + \rho_{ht} + \eta_{ht} + \eta_{hd} + \sigma_3 + \mu_h) I_{dt}\right) + \left(\frac{I_{mdt} - I_{mdt}^*}{I_{mdt}}\right)\left((\rho_{hm} + \rho_{hd} + \rho_{ht} + \eta_{hm} + \eta_{ht} + \eta_{hd} + \sigma_4 + \mu_h) I_{mdt}\right) + \left(\frac{R - R^*}{R}\right)(\delta R) + \left(\frac{S_v - S_v^*}{S_v}\right)\left((\beta_1 + \beta_2 + \mu_v) S_v\right) + \left(\frac{I_{vm} - I_{vm}^*}{I_{vm}}\right)(\beta_1 S_v - \mu_v I_{vm}) + \left(\frac{I_{vd} - I_{vd}^*}{I_{vd}}\right)(\mu_v I_{vd}) + \left(\frac{W - W^*}{W}\right)(\mu_b W), \quad (29)$$

Therefore, if $P_1 < P_2$, then $\frac{dV}{dt} < 0$, and $\frac{dV}{dt} = 0$, if and only if

$$S_h = S_h^*,\ I_{hm} = I_{hm}^*, I_{hd} = I_{hd}^*,\ I_{ht} = I_{ht}^*,\ I_{md} = I_{md}^*,\ I_{mt} = I_{mt}^*, I_{dt} = I_{dt}^*, I_{mdt} = I_{mdt}^*, \mathrm{R} = R^*, S_v = S_v^*, I_{vm} = I_{vm}^*, I_{vd} = I_{vd}^* \text{ and } W = W^*, \quad (30)$$

Therefore the largest invariant set in

$$\left\{S_h^*, I_{hm}^*, I_{hd}^*, I_{ht}^*, I_{md}^*, I_{mt}^*, I_{dt}^*, I_{mdt}^*, R^*, S_v^*, I_{vm}^*, I_{vd}^*, W^* \in \Omega : \frac{dV}{dt} = 0\right\}, \quad (31)$$

is just the singleton set of ε^*, where ε^* is the endemic equilibrium point.

According to Lasalle's Invariant Principle, it, therefore, means that ε^* is globally asymptotically stable in Ω if $P_1 < P_2$.

Sensitivity Analysis: In this section, sensitivity analysis is carried out to identify the most influential parameter(s) on the reproduction number. The techniques in (Akanni and Adediipo, 2018) are applied. Given a parameter, say ξ, the sensitivity index of R_0 with respect to ξ is given by:

$$K_\xi^{R_0} = \frac{\partial R_0}{\partial \xi}\frac{\xi}{R_0}, \quad (32)$$

Where R_0 is defined as:

$$R_0 = \max\left\{\sqrt{\frac{\Lambda_v \mu_h b_m^2 \vartheta_m^2}{(\rho_{hm} + \eta_{hm} + \mu_h)\Lambda \mu_v^2}}, \sqrt{\frac{\Lambda_v \mu_h b_d^2 \vartheta_d^2}{(\rho_{hd} + \eta_{hd} + \mu_h)\Lambda \mu_v^2}}, \frac{v\Lambda\sigma_1}{(\rho_{ht} + \eta_{ht} + \sigma_1 + \mu_h)\mu_b \mu_h}\right\}. \quad (33)$$

In sensitivity analysis, parameters with positive indices contribute to an escalation of infections within the community. Conversely, parameters bearing negative sensitivity indices emerge as potential targets for controlling the spread of diseases in the community, as an elevation in their values correlates with a reduction in the reproduction number. The detailed results in (32) are presented in Table 4 and visually depicted in Figure 1. The sensitivity analysis, based on the initial values from Table 3, illuminates that the most influential parameters for the basic reproduction of malaria are b_m, ϑ_m, and μ_v. This underscores the strategic focus needed to mitigate the triple infection where malaria is dominant,

emphasizing interventions aimed at diminishing the malaria vector population. Similarly, for the basic reproduction number of dengue, the most influential parameters are μ_h, ϑ_d, and μ_v. This suggests that effective strategies to counteract triple infection with dengue as the dominant disease should concentrate on reducing the dengue vector population. Lastly, in the context of the basic reproduction of typhoid, the most sensitive parameters are μ_h, and μ_b. This implies that, to curtail the spread of triple infection where typhoid prevails, one should prioritize minimizing exposure to contaminated water and food while concurrently reducing the discharge of typhoid bacteria into the environment.

Table 3. Parameter Values

Symbols	Values	Sources
μ_h	0.00004	Nthiiri, 2017
μ_v	0.033	Nthiiri, 2017
μ_b	0.645	Nthiiri, 2017
b_m	0.15096	Nthiiri, 2017
b_d	0.000451	Estimated
ϑ_m	12	Nthiiri, 2017
ϑ_d	0.5	Estimated
v	1.37×10^{-9}	Atokolo, W and Omale, D. 2018
ρ_{hm}	0.038	Atokolo, W and Omale, D. 2018
ρ_{hd}	0.1428	Estimated
ρ_{ht}	0.0657	Atokolo, W and Omale, D. 2018
η_{hm}	0.0019	Atokolo, W and Omale, D. 2018
η_{hd}	0.0015	Estimated
η_{ht}	0.002	Atokolo, W and Omale, D. 2018
σ_1	10	Atokolo, W and Omale, D. 2018

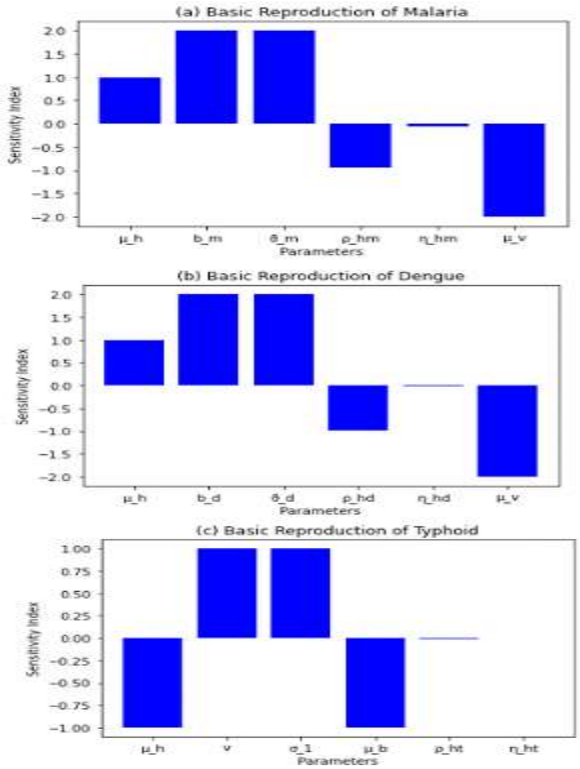

Fig 1. Sensitivity Indices of the Parameters

Table 4. Sensitivity Indices

Parameter	Sensitivity Index
Basic Reproduction of Malaria	
μ_h	-0.9989984972
b_m	2
ϑ_m	2
ρ_{hm}	-0.9514271407
η_{hm}	-0.04757135704
μ_v	2
Basic Reproduction of Dengue	
μ_h	-0.9997228769
b_d	2
ϑ_d	2
ρ_{hd}	-0.9893307467
η_{hd}	-0.01039212969
μ_v	2
Basic Reproduction of Typhoid	
μ_h	1
v	1
σ_1	1
μ_b	-1
ρ_{ht}	- 0.006525794270
η_{ht}	- 0.0001986543157

Conclusion: In this study, we conducted a global stability analysis of the endemic point of the malaria, dengue, and typhoid triple infection model, as well as a sensitivity analysis. The results indicate that the diseases can be eradicated or kept at low levels, regardless of the population. Furthermore, the findings suggest that strategies to reduce malaria and dengue fever vectors should be prioritized to curb the spread of these diseases. Additionally, minimizing exposure to contaminated water and food, as well as reducing the discharge of typhoid bacteria into the environment, can help to reduce or curb the spread of typhoid in the environment.

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