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Schrodinger Equation for Momentum Indicator in the Stock Market

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ABSTRACT: *This paper is a contribution to the application of quantum finance theory. Some of the common momentum indicators include: the rate of change (or the ROC), the relative strengthindex (or the RSI), the moving average convergence divergence (or the MACD) and the stochastic indicator. It is noted that many of the known methods for computing the indicators including those not mentioned here, have mainly concentrated on the historical stock's data spanning over weeks and months, and even at that, there is no empirical basis for most of those methods as they were mere mathematical manipulations and conjecture. Using the model based on the Schrodinger equation for the harmonic oscillator, we developed a method to compute the velocity and momentum of stock prices in a stock market. This offered a proven approach that would give financial technical analysts credible computational method using daily/current stock market data that would improve their quality of advice to potential investors and interested stakeholders. Some randomly selected equities traded on the floor of the Nigeria Stock Exchange were used as our case study.*

Keywords –*Schrodinger equation, quantum finance, velocity indicator, momentum indicator, technical analyst*

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I. INTRODUCTION

Stock markets all over the world have been playing important role in the financial system of any economy. It provides a significant window for investors to raise relatively cheap and long-term capital for businesses and institutions. In view of this strategic role in the economy of nations and its viability, various stakeholders such as equity traders, listed companies as well as regulatory institutions are very much concerned with price behaviours in the stock market, and how to understand and measure the volatility. In particular, equity investors want to be able to gauge their risk and profitability for trading on a particular stock.

This necessity has open up various research activities in stock price behaviour; and because of its stochastic nature, has provided a basis for the entrants of interdisciplinary researchers from the fields of physics and mathematics which now forms the field of Quantum finance – the subject area of this paper. Previous research work in Quantum finance can be traced to the year 1900 when Louis Bachelier published a paper titled "The Theory of Speculation" [1]. Bachelier's paper now famously known as the "Random Walk theory" postulates that a security's current market value is a product of chance rather than the sum of past events or the result of patterns in human behaviour [2].

A major breakthrough in quantum finance came in 1973 with the publication of Fischer Black and Myron Scholes': "The Pricing of Options and Corporate Liabilities" [3], and Robert Merton's: "On the Pricing of Corporate Debt: the Risk Structure of Interest Rates" [4]. Examples of growing recent research activities in financial markets include, "A quantum model for the stock market" by Chao Zhang and Lu Huang (2010), which uses several fundamental assumptions of quantum theory to build a quantum finance model for stock market that quantitatively describe and discuss the distribution of the rate of return and the evolution of the average rate of return over time; "Quantum diffusion of prices and profits" by Edward W. Piotrowski and Jan Sladkowski (2003) – shows that traders tactics can interfere as waves do, and trader's strategies can be reproduced from the corresponding Wigner functions. Others include "Quantum spatial-periodic harmonic model for daily price-limited stock markets" by Xiangyi Menga, Jian-Wei Zhangb, Jingjing Xuc, and Hong Guoa (2004). Their research investigates the behaviour of stocks in daily price-limited stock markets by proposing a quantum spatial-periodic harmonic model, and found that when the trading volume is small, the stock price behaves like a harmonic oscillator, while a large trading volume implies a free particle behaviour. “A Quantum Finance Model for Technical Analysis in the Stock Market” by Ohwadua, E.O. et al [5], developed a model for estimating the expected rate of returns and uncertainties in the daily prices of stocks.

As a follow up, this paper prescribes a method for the estimation of expected velocity and momentum for daily traded stocks in the floor of a stock exchange using the Nigeria Stock Exchange (NSE) as our case study.

II. TECHNICAL INDICATORS FOR THE STOCK MARKET

Technical analysis of stocks is the forecasting of financial price fluctuations based on the examination of past market performance in order to predict future market behaviour [6].Technical analysis uses a variety of different tools and technical indicators to observe – in depth, the effects of market action on stock price for the purpose of forecasting future price trends [7]. The term “market action” includes principal sources of available information such as price and volume. Although this approach has its drawbacks, it offers a special perspective that might be more relevant to stock investment, because stock prices sometimes rise and fall completely independently of the company's operations [8].

Technical analysis may appear complicated on the surface, but it scales down to an analysis of demand and supply in the market in order to determine where the price trend is headed [9]. In other words, technical analysis attempts to use charts and other tools to identify patterns that can be used as a basis for investment decisions rather than measure a security’s intrinsic value as obtained in fundamental analysis. If you understand the benefits and limitations of technical analysis, it can give you a new set of tools or skills that will enable traders to be a better investor over the long-run.

The speed at which the price of stocks moves, up or down, over a given period of time may be referred to as price velocity[10]. If the equity price on any trading day or over a period of time is continuously going up or down at faster and faster velocities, then the price is accelerating, while it is decelerating if the price is moving up or down at slower and slower velocities. On the other hand, the price has constant velocity or zero acceleration if it is moving along a horizontal line, and if the displacement is sideways – which is a special case of horizontal movement, it has zero velocity and zero acceleration.

On the other hand, momentum measures the velocity of price changes whether upward or downward as opposed to the actual price levels themselves. Momentum indicators are used to monitor the underlying "health" of the trend of a particular stock[11]. This monitoring may be achieved through a variety of measurement, and most commonly by assessing the rate at which a stock or financial instrument is rising or falling. Changes in the rate of advance/decline are useful in determining the level of investor enthusiasm – for instance, an uptrend – losing momentum implies that investors are no longer prepared to buy as much stock at current prices; demand pull had run out of steam and we could reasonably expect a period of consolidation before confidence returns.

The idea of trend is perhaps one important concept in technical analysis – a trend may be described as the general direction in which a security or market is headed. An uptrend refers to a series of higher highs and higher lows, while a downtrend consists of lower lows and lower highs. Sideways or flat trends occur when there is little noticeable movement up or down in the peaks and troughs of a trend – or it may even be described as the absence of a well-defined trend in either direction [9].

Some of the common momentum indicators include: the rate of change (or the ROC), the relative strengthindex (or the RSI), the moving average convergence divergence (or the MACD) and the stochastic indicator.

III. SCHRODINGER WAVE EQUATION FOR THE HARMONIC OSCILLATOR

The quantum harmonic oscillator can serve as a mathematical model for a variety of motions, in which the stochastic movement of stock price is being considered. The motion demonstrates a single-frequency traveling wave which takes the form of a sine, cosine, or exponential wave as a function of time or distance. The mathematical form for the harmonic wave motion is given as $\Psi(x,t)$. This wave function, contains the probabilistic properties of the behaviour of given particle, which is referred to here as the stock price. With the aid of the Schrodinger equation, the wave function for the harmonic oscillator is developed which form the basis for the quantum model developed as contained in the paper – “A Quantum Finance Model for Technical Analysis in the Stock Market” from where the momentum and velocity indicators of stock prices can be measured in a daily price limited stock market. The Schrodinger wave equation for the quantum harmonic oscillator is given as:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{dx^2} + V(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}, \quad (1)$$

which is a one-dimensional time-dependent Schrodinger wave equation, where, $\hbar^2 \approx 6.6260693 \times 10^{-34} J \times sec$ or $6.6260693 \times 10^{-34} m^2$ kg / s is the Plank's constant, m is the mass of particle, $\Psi(x,t)$ is the wave function, $V(x) = \frac{1}{2} m\omega^2 x^2$ is the potential energy of particle, and E is the total energy of particle.

3.1 The Probability Interpretation of the Wave Function

Since the wave function contains the probabilistic properties of the behaviour of given particle, we can use the probability distribution to calculate a number of quantities that typically arise in the analysis of random data – the expected value and standard deviation. In the same vein, a particle described by the wave function $\Psi(x,t)$, can be used to determine the expected value of the velocity $\langle v \rangle$of the particle as follows [12].:

$$\langle v \rangle = \frac{d}{dt}\int_{-\infty}^{+\infty} x|\Psi(x,t)|^2 dx$$
$$= \int_{-\infty}^{+\infty} x \frac{\partial}{\partial t}|\Psi(x,t)|^2 dx$$
$$= \int_{-\infty}^{+\infty} x \left[\frac{\partial \Psi^*(x,t)}{\partial t}\Psi(x,t) + \Psi^*(x,t)\frac{\partial \Psi(x,t)}{\partial t}\right] dx \qquad (2)$$

Where $\Psi^*(x,t)$ is the complex conjugate of the wave function $\Psi(x,t)$.
We also can obtain the expected value of the momentum $\langle p \rangle$ as follows[12]:

$$\langle p \rangle = 2mRe\left[\int_{-\infty}^{+\infty} x\Psi^*(x,t)\frac{\partial \Psi(x,t)}{\partial t} dx\right] \qquad (3)$$

IV. DAILY PRICE-LIMITED STOCK MARKET

In the Nigeria Stock Exchange (NSE) Rule Book, Rule 15.29 [13] states that securities shall trade in price increments of one kobo (10^{-2} Naira) and the price movements and price limits on any given trading day shall be $\pm 10\%$. In other words, the rate of return in any trading day cannot exceed $\pm 10\%$ in relation to the previous day's closing price. Thus, the fluctuation of the stock price takes place between the price band in a one-dimensional Schrodinger wave equation for the quantum harmonic oscillator under the influence of the potential, $V(x)$.

V. THE PRICE WAVE FUNCTION

In the paper, "A Quantum Finance Model for Technical Indicators in the Stock Market" [5], the real part of price wave function:

$$\Re(\psi_n(r,t)) = A_n cos\left(\frac{1}{\hbar}E_n t\right) sin\left(\frac{n\pi r}{L}\right) = \sqrt{\frac{2}{5L}} cos(E_n t) sin\left(\frac{n\pi r}{L}\right) \qquad (4)$$

was obtained from the time-dependent Schrodinger equation for the harmonic oscillator given below of(1) above. Also, obtained from the model[5], was the probability density function of the stock price given as:

$$P_n(r,t) = |\psi(r,t)_n|^2 = \begin{cases} \left|\sqrt{\frac{2}{5L}} cos(E_n t) sin\left(\frac{n\pi r}{L}\right)\right|^2 ; & 0 < r < L \\ 0; & r < 0, r > L \end{cases} \qquad (5)$$

where E is the allowed energy:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad (6)$$

with $m$and $L = 0.1$, representing the equity capitalisation or price of stock – the maximum daily stock returns or gain/loss on any trading day at the floor of the Nigeria Stock Exchange as contained in section (4) above.
The following tables display the trading activities for randomly selected stocks at the floor of the Nigeria Stock Exchange (NSE):

Table 1: Sample of 3 equities with daily trading data for 03/07/2017 on the floor of the NSE [5].

STOCKS	PCLOSE m1	OPEN	HIGH	LOW	CLOSE m2	CHANGE	% CHANGE	DEALS	VOLUME	VALUE
AIICO	0.60	0.60	0.62	0.59	0.62	0.02	3.33	25	1,538,420	933,198
NB	161.00	159.99	161.05	159.99	161.05	0.05	0.03	91	420,750	67,479,195
MOBIL	250.02	237.53	237.53	237.53	237.53	-12.49	-5.00	14	24,385	5,795,468

Table 2: Sample of four equities with daily trading data for 08/11/2017 on the floor of the NSE [5].

STOCKS	PCLOSE m1	OPEN	HIGH	LOW	CLOSE m2	CHANGE	% CHANGE	DEALS	VOLUME	VALUE
ACCESS	10.10	10.07	10.14	10.01	10.01	-0.09	-0.89	156.00	4,988,656	50,329,342.73
DSUGAR	15.50	15.83	15.66	15.21	15.46	-0.04	-0.26	67.00	1,149,138.00	17,766,228.42
GTB	42.03	41.39	42.10	41.39	42.10	0.07	0.17	207.00	5,278,500.00	221,820,122.50
UBA	9.75	9.80	9.84	9.76	9.80	0.05	0.51	133.00	6,726,404.00	65,875,525.02
ZENITH	25.80	25.65	25.65	25.51	25.58	-0.22	-0.85	246.00	6,899,958.00	176,515,085.20

Table 3: Summary results of expected rate of return and investment risk (standard deviation) from Tables 1&2 [5].

STOCK	PCLOSE m1	CLOSE m2	RATE OF RETURN *(r)*			ERROR	INV. RISK %
			ARITHMETIC	LOGARITHMIC	MODEL	|LOGARITHMIC – MODEL|	
AIICO	0.60	0.62	0.0333	0.0328	0.0317	0.0011	2.83
NB	161.00	161.05	0.0003	0.0003	0.0003	0.0000	0.02
MOBIL	250.02	237.53	-0.0500	-0.0512	-0.0498	0.0014	3.29
ACCESS	10.10	10.07	-0.0030	-0.0030	-0.0029	0.0001	0.21
DSUGAR	15.50	15.83	0.0213	0.0211	0.0217	0.0006	1.66
GTB	42.03	41.39	-0.0152	-0.0153	-0.0145	0.0008	1.02
UBA	9.75	9.80	0.0051	0.0051	0.0051	0.0000	0.37
ZENITH	25.80	25.65	-0.0058	-0.0058	-0.0056	0.0.0002	0.40

Key

STOCK: List of three equities traded on the floor of the NSE on 03/07/2017; **PCLOSE (m1):** The closing price of stock on the previous trading day of 02/07/2017; **CLOSE (m2):** The closing price of stock on the floor of the NSE on 03/07/2017; **RATE OF RETURN (r)**: The returns – profit or loss on the stock on the floor of the NSE on 03/07/2017; **ARITHMETIC:** Computed arithmetic rate of return of stock; **LOGARITHMIC**: Logarithmic rate of return stock; **MODEL:** Rate of return of stock obtained using the Model;
ERROR $= \mathrm{abs}\ (\mathrm{LOGARITHMIC} - \mathrm{MODEL})$; **INV. RISK:** Investment risk or standard deviation of stock obtained using the Model.

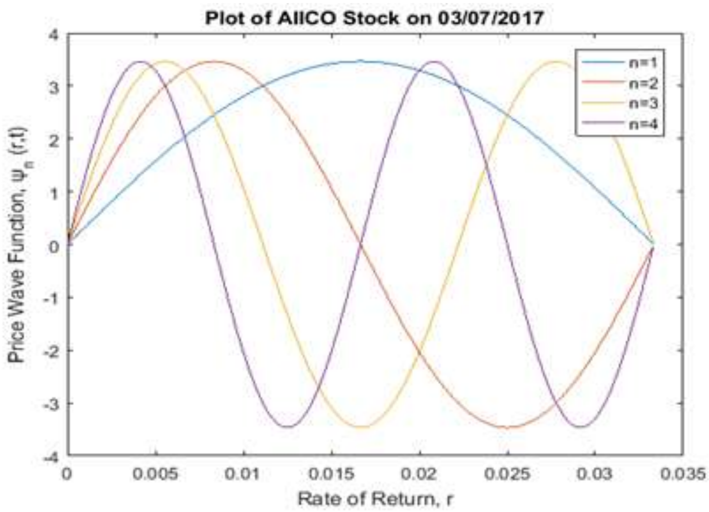
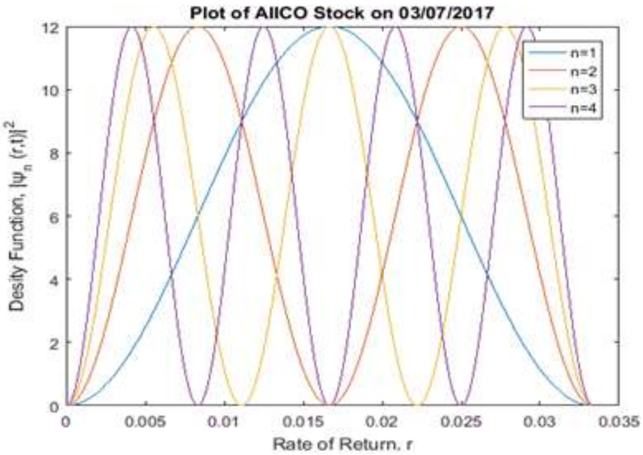


Figure 1: The graph of price wave function, ψ_n (r,t) vs r for AIICO stock when t=0 and n=1,2,3,4[5].



5.1 Velocity and Momentum Indicators from the Price Wave Function

Applying (2) and (3), we shall use the price wave function and the price distribution function in (4) and (5) above to obtain the expected value of the velocity and momentum of stocks whose data is displayed in Tables 1 and 2.

5.1.1 Expected Value of Velocity of stock Prices

From (2), the expected value of the velocity of stock price is given as:

$$\langle v\rangle = \int_{-\infty}^{+\infty} x\left[\frac{\partial\Psi^*(x,t)}{\partial t}\Psi(x,t)+\Psi^*(x,t)\frac{\partial\Psi(x,t)}{\partial t}\right]dx$$

where $\Psi^*(x,t)$ is the complex conjugate of the wave function $\Psi(x,t)$.Thus, given the price wave function, we have the expected value of the rate of return of stock to be:

$$\langle v(t)\rangle = -2{A_n}^2E_n\cos(E_nt)\sin(E_nt)\int_0^L x\sin^2\left(\frac{n\pi x}{L}\right)dx \qquad (7)$$

where A_n is a constant. When $n = 1$, at the ground state of the harmonic oscillator, we have:

$$\langle v(t)\rangle = -2\left(\frac{2}{5L}\right)E_1\cos(E_1t)\sin(E_1t)\int_0^d r\sin^2\left(\frac{\pi r}{d}\right)dr \qquad (8)$$

where $d = \frac{m2-m1}{m1}$ and $m = m2 - m1$; r is the rate of return and $E_1 = \frac{\pi^2}{2md^2}$.

5.1.2 Expected Value of Momentum Stock Prices

From (3), the expected value of the momentum of stock price is given as:

$$\langle p\rangle = 2mRe\left[\int_{-\infty}^{+\infty} x\Psi^*(x,t)\frac{\partial\Psi(x,t)}{\partial t}dx\right]$$
$$= 2m\Re\left[\int_0^L x\,A_n\cos(E_nt)sin\left(\frac{n\pi x}{L}\right)\frac{\partial\Psi(x,t)}{\partial t}dx\right]$$
$$= -2m{A_n}^2E_n\cos(E_nt)sin(E_nt)\int_0^L x\,sin^2\left(\frac{n\pi x}{L}\right)dx \qquad (9)$$

When $n = 1$, at the ground state of the harmonic oscillator, we have:

$$\langle p(t)\rangle = -2m\left(\frac{2}{5L}\right)E_1\cos(E_1t)sin(E_1t)\int_0^d r\,sin^2\left(\frac{\pi r}{d}\right)dr \qquad (10)$$

5.1.3 Summary of Results for Expected Value of Velocity and Momentum of Stock Prices

Table 4: Summary results of expected velocity and momentum of stocks from Tables 1&2 using (5.1.1) & (5.1.2).

STOCK	PCLOSE m1	CLOSE m2	d $\frac{m2-m1}{m1}$	VELOCITY $\langle v\rangle$	MOMENTUM $\langle p\rangle$
AIICO	0.60	0.62	0.0333	3.9697	14.7404
NB	161.00	161.05	0.0003	17661.0000	1114.9000
MOBIL	250.02	237.53	-0.0500	0.3000	3.7790
ACCESS	10.10	10.01	-0.0089	157.6402	20.4073
DSUGAR	15.50	15.83	0.0213	216.6322	10.6616
GTB	42.03	41.39	-0.0152	35.2421	30.4590
UBA	9.75	9.80	0.0051	1428.7000	68.1631
ZENITH	25.80	25.65	-0.0058	704.6700	79.1879

VI. CONCLUSION

In this paper, we have adopted the price wave function developed in the paper titled: "A Quantum Finance Model for Technical Indicators in the Stock Market" [5], to compute the expected values of velocity and momentum of stock prices for randomly selected stocks from the trading activities of the stocks on the floor of the Nigeria Stock Exchange. The computed values are displayed in table 4 above.

6.1 Interpretation of the Velocity of Stock Price

Recall that we defined the velocity of stock price as the speed at which the price of stocks moves, up or down, over a given period of time. Here, we considered the movement of stocks price on a typical trading day and estimated the average speed from the open time to the closing time of trading activities.

From the Table 4, the velocity of the three stocks – AIICO, NB and MOBIL are given as 3.7790, 17661, and 0.3000 respectively. It is observed that the velocity of NB stock is uncontrollable in view of the fact that the return is very small, less than the 10^{-2}, we therefore omit NB from further analysis. Now, considering the speed between the AIICO and the MOBIL stock, we observe that AIICO stock moves faster than the MOBIL stock. Technical Analysts may have different interpretation for this behaviour –the mass of the MOBIL stock and the percentage of change that took place. So, we conclude that the price of AIICO stock accelerated thanthe price of MOBIL stock. The question is, why do prices of stocks accelerate or decelerate? Well, we have been able to quantitatively measure this stock behaviour, but we shall leave further interpretation to the financial expert or stock Technical Analyst to make use of this data in order to offer a better advice to the potential investors or other stakeholders in the market.

6.2 Momentum of Stock Price

Recall that we defined momentum as a measure of the velocity of price changes whether upward or downward as opposed to the actual price levels themselves - momentum indicators have been used here to measure the underlying health of the trend of the eight stocks in the Table 4 above.

From the Table 4, the momentum or trend for three stocks – AIICO, NB, and MOBIL are given as 14.7404, 1114.9, and 3.7790.We observe that while NB stock is still uncontrollable as indicated earlier, however, AIICO has a better showing than the MOBIL stock. Thus, it is observed that the level of investor enthusiasm for AIICO stock is more than for the MOBIL stock and investors may not be prepared to buy much stock of MOBIL at current prices as demand pull has run out of steam and we could reasonably expect a period of consolidation before confidence returns. All the same, we are not trying to take over the job of financial analyst here, but we believe we have been able to provide the much-needed data for the technical analyst to do its job better.

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